Real Options and Investment in Mobile Networks

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Chapter 1

Introduction

1.1 Investment in mobile networks is inherently subject to irreversibility and uncertainty. These two factors have a crucial impact on the *timing* and *type* of investments that are efficient.

1.2 For some types of investment, recovery of the cost of investment through resale is simply not possible. An example is infrastructure such as masts. A mobile mast has little value outside of the industry. As a result, its resale value is closely correlated to the economic conditions of the industry. So, if conditions turn out to be unfavourable (e.g., because industry-wide demand for mobile services is low), a firm wishing to disinvest by removing and reselling the equipment would find many other firms also wanting to resell. The economic value of the equipment therefore moves up or down with the economic conditions of the industry, making the investment effectively irreversible.

1.3 In addition, a large portion of the initial cost of mobile masts is the cost of erecting the mast. This cost cannot be recovered at all. Further costs will be incurred in recovering the mast for resale; these too cannot be recouped. (In some cases recovery costs will exceed the second-hand value of the mast itself; in this case abandonment, if this can be done at no cost, would be preferable to recovery of the mast.)

1.4 There is considerable uncertainty surrounding mobile investments. Future demand for telecoms services and products, and hence future revenue, is highly uncertain. Uncertainty is greater still for products that are yet to be introduced to the market, for which demand is especially difficult to forecast. Technological innovation also acts to make the returns from an investment unpredictable, as the costs of delivering services may change in the future. Any source of uncertainty affecting future revenues is relevant for investment planning.

1.5 Operators are occasionally faced with a situation where even before recouping their investments in existing infrastructure, they embark on further investment in a new generation of networks. This phenomenon is common in the mobile sector, particularly in the context of 3G services, where the high cost of licensing and equipment have left operators facing substantial investment requirements at the early stage of network deployment.

1.6 The combination of irreversibility and uncertainty creates *real options*. This phrase is intended to reflect two facts. First, prior to making an irreversible investment under uncertainty, a firm holds an "option" on whether and when to invest; once it invests, the firm forfeits the option. Secondly, the option involves *real*, rather than financial assets.

1.7 There is now a considerable literature on the theory and practice of incorporating real options into investment analysis. The earlier literature is summarised well in Dixit and Pindyck (1994). More recent developments can be found in e.g., Schwartz and Trigeorgis (2004). The general lessons from this literature are that the opportunity cost of investing (and so forfeiting a real option) can be very large, and investment rules (such as the classical net present value, or NPV, rule) that ignore it can be grossly in error. Regulation that ignores the opportunity cost will lead to inefficient investment.

1.8 Regulators have been somewhat slower than academics and the business community to allow for real options. In its 1999 report on charges made by Cellnet and Vodafone for terminating calls from fixed-line networks, the Competition Commission stated that

> "[t]he value of the delay option could well be a factor in decision-making in many industries. How far it applies to mobile telephony is less clear. ... We are therefore not persuaded that there is a quantifiable addition that we should make to the cost of capital ..." (para. 2.288).

Real options were not considered in the 2003 report on mobile termination charges. Most recently, Ofcom has recognised in principle that real options should be included in investment decisions:

> "Ofcom concludes that, going forward, its analysis should take account of the value of real options where appropriate."[1]

1.9 Ofcom has suggested that, if real options are to be incorporated, they should be reflected in the *cost of capital* used for calculating long-run incremental costs (LRICs). This is only partially correct; in fact, a proper treatment of the real options that arise in capacity investment requires more than a simple adjustment to the cost of capital.

1.10 In this report, we consider how real options should be included in investment decisions and cost calculations, taking into account the different types of option that arise. The type of option that occurs depends on the nature of the investment decision being undertaken:

1.10.1 When to invest.

1.10.2 How to invest.

1.11 The real option approach broadens the issue of investment beyond the "now-or-never" decision implicit in the NPV rule to include the question of when investment should occur. When the opportunity cost of investment is included in the investment decision, efficient investment typically occurs later (in some sense) than when the opportunity cost is ignored. One consequence of this is that efficient investment occurs at a point when it appears that super-normal profits are earned. A natural (and frequent) policy response is to infer that there is market power, and to regulate to limit the market power and super-normal profits. This response would cause, however, inefficient investment. In order to determine efficient regulation in this situation, it is necessary to include the opportunity cost of the investment. This requires the details of the investment to be specified: the extent to which the cost is sunk; and the nature of uncertainty surrounding the investment. More particularly, the setting of regulated prices for mobile network operators (MNOs) needs to take into account the opportunity cost of investment for the MNOs, as well as the direct capital and operating expenditures. This is true even when the regulated prices are set in order to mimic perfect competition in the sector.

1.12 The real options approach also recognises that investment can itself generate further options, the value of which should also be included

[1] Ofcom, 18 August 2005: "Ofcom's approach to risk in the assessment of the cost of capital—Final Statement." Available at http://www.ofcom.org.uk/consult/condocs/cost_capital2/statement/

in an assessment of the investment. It can be efficient to undertake now a more costly investment, if that investment allows more flexibility in the future should conditions change. In short, flexibility has to be considered in the investment decision; the value of flexibility—or, conversely, the opportunity cost of not having flexibility—has to be included in the investment calculation. For example, it may be efficient for MNOs to build mobile base sites that can be more easily upgraded in the event that demand for 3G services exceeds forecasts. The investment may appear excessive in the base-case scenario of traffic growth. When uncertainty is included explicitly in the calculation, however, the investment can be efficient.

1.13 Our objective in this report is to indicate how the existing LRIC model used by Ofcom can be adapted to take account of real options. In the course of the assessment, we arrive at rough estimates of the quantitative impact of real options. For example, we find that allowing for real options when making network capacity investments increases the LRIC by around 35%. Necessarily, this estimate is approximate. More generally, we establish

- How real options can be taken into account within the broad framework of the existing LRIC model.
- That real options can have a significant quantitative effect on the termination charge that should be applied.
- That real options will increase the effective LRIC of termination of MNOs. The increase arises in order to yield adequate returns on investments that are irreversible and inherently risky, so that investment occurs efficiently.

Chapter 2

The option of when to invest

2.1 In this section, we consider how to incorporate real option factors in the decision of when to invest in a mobile network. We start with a brief review of the current LRIC model; we then suggest how this model could be revised to allow for real options.

2.2 In broad outline, the current LRIC model works as follows:

2.2.1 Demand is forecast: traffic (minutes of voice, bytes of data) and subscribers.

2.2.2 The network is built to meet forecast demand.

2.2.3 Operating and capital expenses are annualised, using the appropriate depreciation method and allowing for cost of capital.

2.2.4 Costs are allocated to services.

2.2.5 Mark-ups are applied to the estimated LRIC of the service to recover fixed and common costs, plus other factors (such as network externalities).

2.3 The calculation of LRIC is intrinsically forward-looking. This means that forecasts play a key role in the calculation. Forecasts are, of course, inherently uncertain. To a limited extent, the current model allows for forecast uncertainty. Three different scenarios (low, medium and high) are calculated for different aspects of demand (e.g., average voice traffic per subscriber; outgoing messages per average subscriber).

2.4 The current LRIC model incorporates the irreversibility of investment though its depreciation assumptions. LRIC uses forward-looking costs, setting rates based on current or expected future equipment costs, rather than the historical costs actually paid for the equipment.

2.5 So, the current LRIC model includes, to a limited extent, both uncertainty and irreversibility. It does not, however, allow for *interaction* between the two. And so it does not incorporate the real options that arise because of the combination of uncertainty and irreversibility.

2.6 In order to demonstrate how to adapt the LRIC model, and to assess the effect of incorporating real options, we use a simplified model of mobile pricing and investment decisions. Our model has the following features:

  2.6.1 The degree of uncertainty is explicitly parameterised, rather than being captured partially in alternative scenarios.

  2.6.2 The degree of irreversibility is explicitly parameterised, rather than being captured partially by depreciation profiles.

  2.6.3 Investment takes place in many stages, rather than once-and-for-all.

2.7 We present two versions of the model. The first version is a simplified version of the current model; the aim is to provide a benchmark against which we can measure the effect of including real options. The second version allows for uncertainty and irreversibility.

2.8 Following the current LRIC model, in both versions of the model, we concentrate on *traffic* as the main driver of investment decisions and costs; we ignore changes in the number of subscribers. The main justification for this assumption is to simplify the analysis. Not too much is lost by the assumption, however, since (according to the LRIC model) around 90% of total costs can be attributed to traffic.

2.9 There are five traffic flows that are relevant for an individual MNO.

  2.9.1 on-net: originates and terminates on the same network. Denote the volume of this traffic (in minutes) by x.

  2.9.2 off-net to fixed: originating traffic that terminates on a fixed network. Denote this traffic volume by $x_{\to F}$.

  2.9.3 off-net to mobile: originating traffic that terminates on a mobile network. Denote this traffic volume by $x_{\to M}$.

  2.9.4 from fixed: terminating traffic that originates on a fixed network. Denote this traffic volume by $x_{F\to}$.

  2.9.5 from mobile: terminating traffic that originates on a mobile network. Denote this traffic volume by $x_{M\to}$.

2.10 We make the following assumptions about the balance of traffic:

2.10.1 Traffic between mobile networks is balanced: $x_{\to M} = x_{M\to}$, which we relabel as x_M. While this is not completely correct, it is not a bad approximation of the current situation.

2.10.2 Traffic between fixed and mobile networks is imbalanced, with the traffic volume from mobiles to fixed being roughly 1.5 times the volume from fixed to mobile. (The factor of 1.5 is taken from the 2003 Competition Commission report on mobile termination.)[1] If $x_{F\to}$ is relabelled as x_F, this means that $x_{\to F} \approx 1.5x_F$.

2.11 The total traffic terminating on the MNO is $X = x + x_M + x_F$. Due to our traffic balance assumptions, the total originating traffic is $x + x_M + 1.5x_F$. We assume that the proportions of terminating traffic are constant over time: that a proportion a is on-net, b is off-net to mobile, and the remainder $1 - a - b$ is off-net to fixed. Hence we assume that

$$\frac{x}{x + x_M + x_F} = a, \quad \frac{x_M}{x + x_M + x_F} = b.$$

(The current LRIC model uses approximately the same assumption: in the medium term, a is roughly 30% and b is roughly 20%.) The fraction of total traffic arising from termination is then

$$\frac{2}{5 - a - b}.$$

2.1 Investment under certainty: a benchmark

2.12 In this benchmark version of the model, X can change over time, but does so *deterministically* i.e., there is no uncertainty about traffic forecasts.

2.13 The variable (annual) profit of an MNO from origination is

$$\pi^O = (p - c^O)(x_M + x + 1.5x_F) - \bar{t}x_M - 1.5fx_F. \qquad (2.1)$$

In this expression, p is the retail price charged per minute by the MNO. We are ignoring, therefore, non-linear pricing. c^O is the marginal cost

[1]In table 5.20 in Chapter 5, the Competition Commission state traffic volumes for the year 2001/2002 to be 23.26 billion minutes mobile to fixed, and 15.22 billion minutes fixed to mobile.

of origination, assumed to be constant. $\bar{t}$ is the regulated termination charge for traffic between mobiles. f is the termination charge for traffic that terminates on fixed networks.

The variable (annual) profit of an MNO from termination is

$$\pi^T = (\bar{t} - c^T)(x_M + x_F) - c^T x \tag{2.2}$$

where c^T is the marginal cost of termination, assumed to be constant. Hence the total variable profit of an MNO, which is the sum of variable profits from origination and termination, is

$$\begin{aligned}\pi = \pi^O + \pi^T &= (x + x_M)(p - c^O - c^T) + x_F(1.5(p - c^O - f) + \bar{t} - c^T) \\ &= X\left[p - c^O - c^T + \left(\frac{1 - a - b}{2}\right)(p + 2\bar{t} - 3f - c^O)\right]. \end{aligned} \tag{2.3}$$

The MNO therefore earns a margin of $p - c^O - c^T$ (the retail price minus the total marginal cost of origination plus termination) on on-net traffic, and off-net traffic between mobile networks. On off-net traffic to and from fixed networks, the MNO earns a margin that has two parts: from origination, which earns a margin of $p - c^O - f$; and from termination, with a margin of $\bar{t} - c^T$.

2.14 The variables c^O, c^T, f and p are *parameters* in our analysis. c^O and c^T are determined by mobile technologies. f is determined (predominantly) by regulation of the major fixed network. p is determined by competition in the mobile retail market. Our focus is on $\bar{t}$, set by the regulator according to the amount of capacity investment.

2.15 The total volume of traffic carried by the MNO (the sum of originating and terminating traffic) is $(5 - a - b)X/2$, and hence is proportional to the volume of terminating traffic X. In order to carry this total volume of traffic, investment in network capacity is required. In this first version of the model, we suppose that capacity investment occurs to match exactly (forecast) traffic volumes. That is, we do not allow for the possibility of delayed investment. This is entirely reasonable in a model with no uncertainty. To make the model as straightforward as possible, suppose that network capacity is measured in minutes of traffic.

2.16 The actual calculation of LRIC is complicated, but the basic idea is straightforward: the investing firm is reimbursed for its capital cost through an annual annuity payment that reflects the firm's cost of capital. The LRIC also includes payment for one-time set-up costs and direct and indirect ongoing fixed costs, which we ignore (for simplicity). We also assume that capacity utilisation and MEA prices are constant, for simplicity. Let the cost of capital be ρ; and the life-time of assets be T (e.g., 15–20 years). Let the unit cost of capacity be κ.[2] Then the LRIC of an amount of capital K is

$$\frac{\rho(1+\rho)^T}{(1+\rho)^T-1}\kappa K \equiv \delta\kappa K$$

where

$$\delta \equiv \frac{\rho(1+\rho)^T}{(1+\rho)^T-1}.$$

For example, if the cost of capital is 15% and the asset lifetime is 15 years, then $\delta = 0.171$. If capital is infinitely lived, then $\delta = \rho$.

2.17 Suppose that the regulated termination charge $\bar{t}$ is set equal to the LRIC of the installed capacity required for termination. When the volume of terminating traffic is X, the LRIC of termination is $\delta\kappa X$. The termination charge per unit of traffic is then

$$\bar{t} = \delta\kappa. \tag{2.4}$$

2.18 This completes the first version of the model. Traffic volumes, and particularly termination volumes X, are the primary drivers. Investment occurs in order to carry the traffic volumes. The termination charge $\bar{t}$ is set at the LRIC of invested capacity relating to termination.

2.2 Investment under uncertainty

2.19 We now suppose that there is uncertainty in demand forecasts. Specifically, the traffic volume X varies stochastically. Let the volume of termination traffic at time t be X_t. Following e.g., Dixit and Pindyck (1994), suppose that X_t follows a Geometric Brownian Motion (GBM):

$$dX_t = \mu X_t dt + \sigma X_t dz_t \tag{2.5}$$

where dz_t is the increment of a Wiener process. Hence *percentage changes* in traffic volumes are assumed to follow a *Normal distribution.*

[2] κ can be viewed as an exchange rate, to 'convert' capacity expressed in minutes to pence. The units of κ are therefore pence per minutes (ppm).

$\mu \geq 0$ is known as the *drift* parameter; on average, traffic volumes increase in percentage terms at a rate μ. $\sigma \geq 0$ is the volatility parameter, and measures the degree of uncertainty. If σ is zero, then we are back to the first version of the model, in which there was no uncertainty. A higher σ corresponds to higher levels of uncertainty. (We discuss below how the drift and volatility parameters, μ and σ, can be estimated.)

2.20 In this second version of the model, we do not assume that capacity investment occurs to ensure that traffic volumes can be carried. Instead, we allow the MNO to delay investment until traffic volumes are 'sufficiently' high. We shall determine the extent to which the MNO delays investment; and how the extent of delay depends on the various parameters—in particular, σ.

2.21 An immediate implication of delayed investment is that the MNO can carry the lower of actual traffic volumes, X_t, and the capacity of its network. If X_t rises above the network's capacity K, say, and no investment occurs, then the network is able to carry only K minutes of traffic. This is a simplifying assumption used mostly for convenience. We do not think that an MNO will actually refuse to serve traffic over its capacity K. In practice, as traffic volumes increase for a fixed capacity, the traffic will be carried but service quality (such as bandwidth and blocking probabilities) will degrade. We capture this in this simple model by supposing that volume over capacity is not served at all.

2.22 The MNO's variable (flow) profit is

$$\begin{aligned}\pi &= \left[p - c^O - c^T + \left(\frac{1-a-b}{2}\right)(p + 2\bar{t} - 3f - c^O)\right] \min\{X_t, K\} \\ &\equiv \theta \min\{X_t, K\}, \qquad (2.6)\end{aligned}$$

where $\theta = p - c^O - c^T + (1-a-b)(p + 2\bar{t} - 3f - c^O)/2$ is the margin per unit.

2.23 There are three important regions to consider.

2.23.1 Region 1: X_t is less than the MNO's capacity; hence no investment is necessary.

2.23.2 Region 2: X_t is greater than the MNO's capacity; but is not sufficiently high to trigger investment by the MNO.

2.23.3 Region 3: X_t is greater than the MNO's capacity, and is high enough to trigger investment.

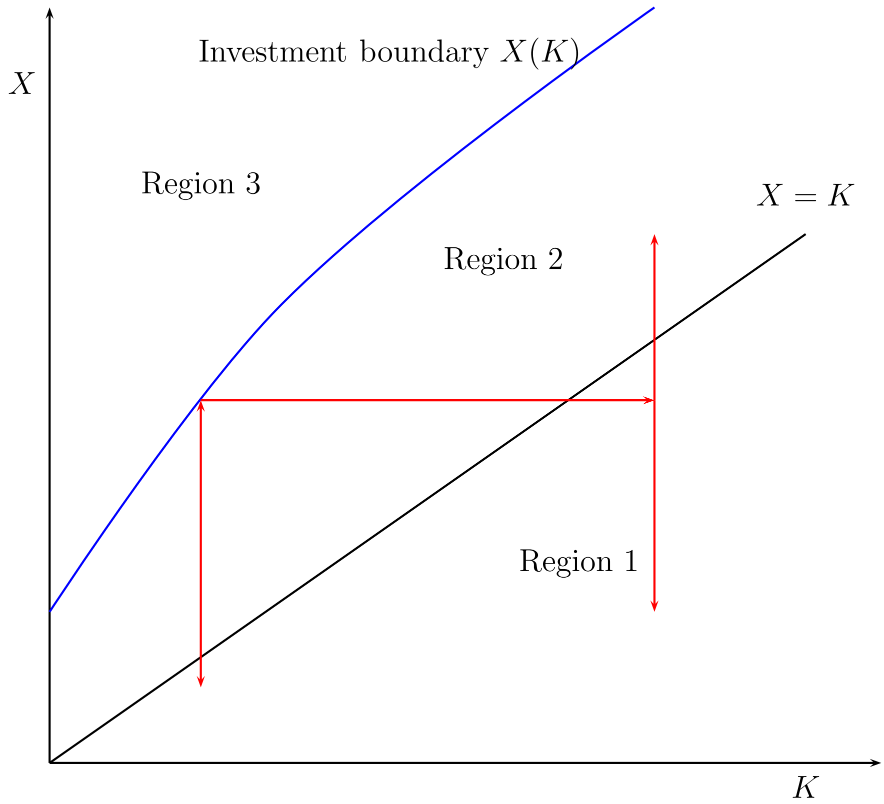

Figure 2.1: Investment and non-investment regions

2.24 We illustrate these regions in figure 2.1. The network's capacity is shown on the horizontal axis; the current traffic volume X_t is shown on the vertical axis. The vertical lines in the figure illustrate how X_t changes over time, for a given level of network capacity. Of course X_t may go down as well as up. Eventually, however, it will rise to hit the third region (having passed through the second region). At this point, the network invests: movement occurs according to the horizontal arrows that are shown.

2.25 The task now is to determine the boundary of the third region, at which investment is triggered. The current value (i.e., the expected present discounted value of future profits) of the MNO depends on two factors: the current traffic volume, X_t; and current capacity K. So, we denote the current value of the MNO by $V(X_t, K)$. It is straightforward to show (see e.g., Dixit and Pindyck (1994)) that $V(X_t, K)$ must satisfy the following differential equation for combinations of X_t and K in regions 1 and 2:

$$\frac{1}{2}\sigma^2 X_t^2 \frac{\partial^2 V(X_t, K)}{\partial X_t^2} + \mu X_t \frac{\partial V(X_t, K)}{\partial X_t} - \rho V(X_t, K) + \theta \min\{X_t, K\} \tag{2.7}$$

where ρ is the cost of capital.

2.26 This differential equation can be solved to give the general solution

$$V(X_t, K) = \begin{cases} A(K)X^\beta + \frac{\theta}{\rho-\mu}X_t & X_t < K, \\ A(K)X^\beta + \frac{\theta}{\rho}K & X_t \geq K. \end{cases} \tag{2.8}$$

The firm's value therefore has two components. The first component, $A(K)X^\beta$, is an option value anticipating future capacity investment. The second component is the expected net present value of revenue from the MNO's current capacity. If current capacity exceeds current traffic, this is $X/(\rho - \mu)$; otherwise, it equals K/ρ.

$A(K)$ is a coefficient which is not of primary interest. β is a constant, given by

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{2.9}$$

2.27 We now determine the investment boundary between regions 2 and 3. As soon as X_t hits this boundary, the MNO invests, increasing its capital from K to K'. Since investment is irreversible, $K' \geq K$. Immediately after investment, the firm's value is $V(X_t, K')$. Three conditions must hold.

2.27.1 Since value functions are forward-looking, anticipating future movements in X and investments in capacity, the firm's value just before investment must be equal to its value just after investment. Hence

$$V(X_t, K) = V(X_t, K') - \kappa(K' - K). \tag{2.10}$$

2.27.2 The amount of investment, $K' - K$, is chosen optimally by the MNO, so that the marginal return from investment equals the marginal cost of investment. Hence

$$\frac{\partial V(X_t, K')}{\partial K} = \kappa. \tag{2.11}$$

2.27.3 There is a further, more technical optimality condition, known as *smooth pasting*. This requires not only that the firm's values immediately pre- and post- investment are equal, but also that they are *smoothly* equal. Hence

$$\frac{\partial V(X_t, K)}{\partial X} = \frac{\partial V(X_t, K')}{\partial X}. \tag{2.12}$$

2.28 These three conditions can be solved simultaneously to give the investment boundary, which we denote $X(K)$, and the optimal amount of investment, which we denote $K'(X)$. The investment boundary is given by

$$X(K) = \frac{\beta}{\beta - 1}\left[K + \frac{\rho\kappa}{\theta}(K' - K)\right]. \tag{2.13}$$

Investment does not occur as soon as the traffic volume X hits network capacity K. The expression in equation (2.13) shows the two ways in which this arises. First, uncertainty and irreversibility causes delay. This is reflected in the term $\beta/(\beta - 1) > 1$. As figure 2.2 shows, this term is increasing in the degree of uncertainty, σ: greater uncertainty gives a higher investment threshold, all other things equal. (We return below to what other things may change.)

2.29 Uncertainty therefore seems to appear in a simple way, via the 'markup' term $\hat{\beta} = \beta/(\beta - 1)$. Thus, it might appear that real options can be dealt with simply by adjusting upwards the cost of capital by a factor that, if not equal exactly to $\hat{\beta}$, is at least greater than 1. This might be appropriate when a single investment decision is under consideration. But as equation (2.13) makes clear, it is not appropriate for multi-stage investments—which is the relevant case for MNOs. Instead, there is an additional term,

$$\frac{\rho\kappa}{\theta}(K' - K)$$

which depends on a number of factors. First, investment delay is greater when the following investment is larger (that it, $X(K)$ increases

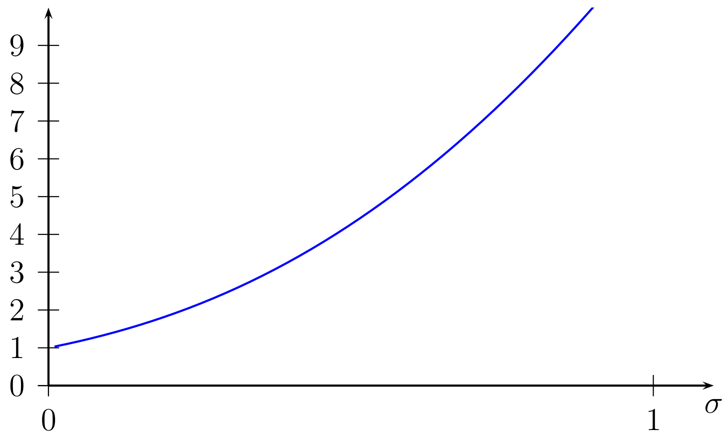

Figure 2.2: Uncertainty factor $\beta/(\beta-1)$

in the size of the investment increment $K' - K$). It is also increasing in the normalised annualised cost of investment. κ is the unit cost of capital. On an annual basis, adjusting for the cost of capital, this cost is $\rho\kappa$. After investment, the MNO earns a return θ on each unit of traffic. Hence the unit cost of investment must be adjusted by the factor $1/\theta$.

2.30 There are a number of possibilities for the amount of investment that occurs.

2.30.1 $K' < X(K)$: investment does not meet the current volume of traffic.

2.30.2 $K' = X(K)$: investment meets exactly the current volume of traffic.

2.30.3 $K' > X(K)$: investment increases capacity above the current volume of traffic.

We shall discuss in more detail the third case (of 'over-investment') in chapter 3. In the appendix, we argue that, in the absence of fixed costs to investment, the optimal amount of investment is $K' = X(K)$: that is, investment occurs so that capacity equals the current traffic volume.

2.31 In this case,

$$X(K) = K' = \frac{\hat{\beta}(1-\phi)}{1-\hat{\beta}\phi} K > K \qquad (2.14)$$

where

$$\hat{\beta} \equiv \frac{\beta}{\beta - 1}, \qquad \phi \equiv \frac{\rho\kappa}{\theta}. \tag{2.15}$$

It is then easy to see that $X(K)$ increases in both

2.31.1 σ, the degree of uncertainty, and

2.31.2 ϕ, the adjusted unit cost of investment.[3]

2.32 The remaining task is to determine the level of the termination charge $\bar{t}$. The termination charge $\bar{t}$ set by the regulator should now reflect the option values inherent in investment. If it does not, then MNOs would invest inefficiently: the regulated charge would not offer sufficient reward for the risk involved in investment, and MNOs would undertake too little investment. Manipulation of equation (2.13) shows that the correct LRIC in this situation is

$$\rho\kappa + \theta - \hat{\beta}\rho\kappa > \rho\kappa.$$

This can be contrasted with the LRIC that would result if uncertainty and real options were ignored: $\rho\kappa$. (The LRIC is $\rho\kappa$ since, implicitly, we have assumed that assets are infinitely-lived. A standard adjustment applies for assets with finite lives.) Hence the LRIC with real options should be increased by an amount

$$\lambda \equiv \theta - \hat{\beta}\rho\kappa.$$

This can be solved for the implied termination charge $\bar{t}$ *per unit of capital*

$$\bar{t} = \frac{p - c^O - c^T + \left(\frac{1-a-b}{2}\right)(p - 3f - c^O) - (\hat{\beta} - 1)\rho\kappa}{a + b}. \tag{2.16}$$

2.33 This completes the second version of the model. Traffic volumes, and particularly termination volumes X, are the primary drivers. Investment occurs whenever traffic volumes are sufficiently high, hitting an investment boundary (which is above the current capacity). The amount of delay is determined by a number of factors, including the degree of uncertainty. The termination charge $\bar{t}$ is set at the LRIC of invested capacity relating to termination (which includes option values).

[3] In order for the model to be consistent, it must be that $\hat{\beta}\phi < 1$.

2.34 We can now compare the benchmark model, with no uncertainty, with the model in which real options are considered. In many ways, the two versions of the model are conceptually very similar, as paragraphs 2.18 and 2.33 indicate. The key difference between the models lies in the investment behaviour of the MNO. In the first version, without uncertainty, investment occurs to match traffic volumes (which are assumed to increase at the trend rate). In the second version, the combination of irreversibility and uncertainty causes the MNO to delay investment until traffic volumes are sufficiently high.

2.35 The investment delay that occurs with uncertainty is *efficient*: it is not something that a regulator should attempt to correct. In fact, the investment pattern in the benchmark model (with no uncertainty) is inefficient when traffic volumes are uncertain.

2.36 Finally, note that the determination of the termination charge $\bar{t}$ in the case of uncertainty requires additional parameters to be used, compared to the 'certainty' termination charge. Equation (2.16) involves not only ρ and κ, the two key parameters for the 'certainty' termination charge, but also p, c^O, f, a and b. The reason is that, with real options, we have to determine the *timing* as well as the *amount* of investment. When traffic volumes change deterministically, the timing of investment is straightforward: subject to the 'lumpiness' of investment, capacity should be expanded to match volume increases. When traffic volumes change stochastically, there is investment delay, as we have shown. In order to determine when investment occurs, we have to consider the returns to investment. That means in turn that we must consider the margin that MNOs earn on each unit of capacity. As a result, the additional parameters appear in the appropriate LRIC of investment, and hence the termination charge.

2.3 Quantifying the effect of uncertainty on the LRIC

2.37 In order to compare the termination charge $\bar{t}$ with and without uncertainty, we have to estimate the relevant parameters. There are ten parameters in the model, summarised in table 2.1. The estimates that we use for the parameters are not intended to be definitive. For example, we use a historical and average figure for the cost of capital. A more comprehensive study would update this estimate using more recent market data. The objective of the exercise is to illustrate how the various parameters would be used to set the LRIC for termination.

Parameter	Meaning	Estimate
p	retail price (ppm)	10
c^O	marginal cost of origination (ppm)	0
c^T	marginal cost of termination (ppm)	0
f	termination charge on fixed networks (ppm)	1
a	on-net traffic proportion	0.3
b	off-net to mobile traffic proportion	0.2
γ	deflator (%)	2.5
ρ	cost of capital (nominal, pre-tax %)	11.3
μ	drift (%)	9.5
σ	volatility	14.1
κ	unit cost of capacity (ppm)	43.5

Table 2.1: Model parameters

2.38 Two crucial parameters for our analysis are μ (the drift rate of traffic volume) and σ (the volatility parameter). There are two sources for estimating these parameters. The first is historical traffic figures. Figure 2.3 shows quarterly percentage changes in total traffic volumes (outgoing plus incoming) for all UK mobile networks over the period July 1993 to July 2001.[4] From these historical figures, the annual drift rate can be estimated at 42.87%, and the annual standard deviation is 7.71%.

2.39 One criticism of this approach is that it may underestimate the true volatility, because it is backward looking and does not account for the uncertainties over the future of mobile growth. It may also overestimate the future growth rate, as the period analysed corresponds to rapid penetration of mobile services from a very low base. Share prices, on the other hand, might provide a more forward-looking estimate of volatility. At issue is how to relate the volatility of share prices to the underlying volatility of traffic volumes.

2.40 Denote the value of an MNO by V. Share price data can be used to

[4]The traffic figures are taken from the April 2002 Analysys model of LRIC for mobile networks.

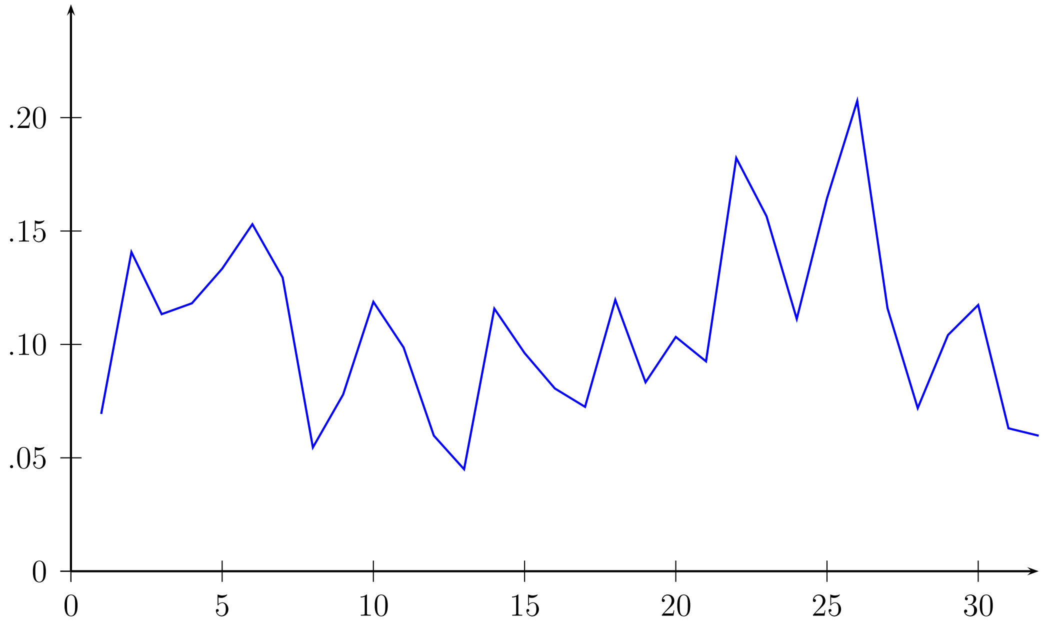

Figure 2.3: Quarterly percentage changes in traffic volumes, July 1993–July 2001

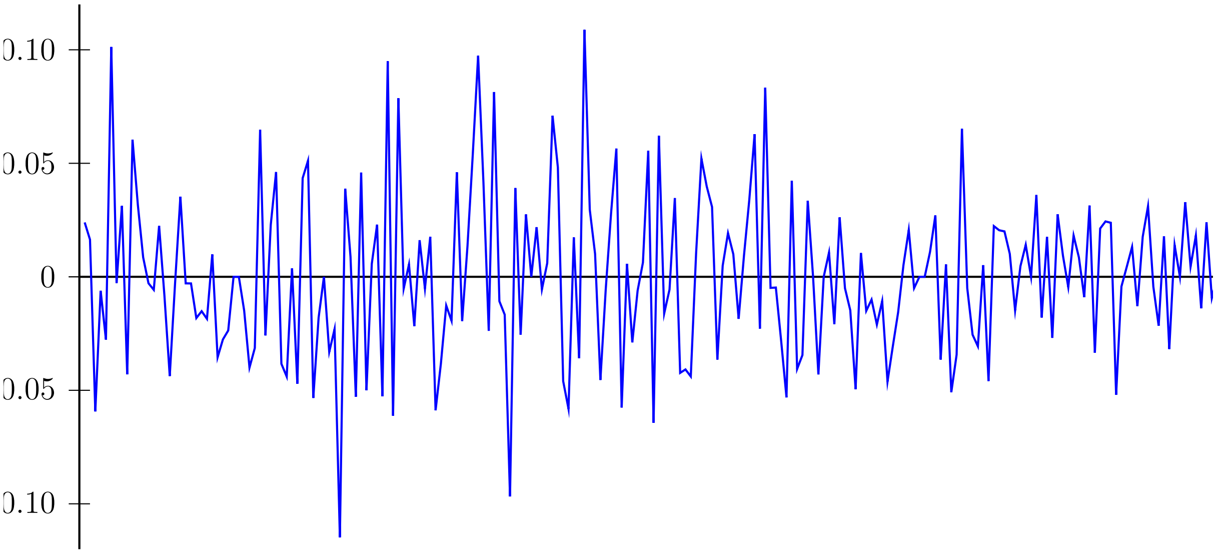

Figure 2.4: Percentage change in O_2 daily share price, 20/11/2001–07/03/2006

estimate the parameters μ_V and σ_V in the equation

$$dV = \mu_V V dt + \sigma_V V dz. \tag{2.17}$$

There are difficulties in isolating returns from UK mobile telephony. The easiest MNO to study for this exercise is, therefore, O_2, for whom the majority of business is in the UK. Daily share price data were obtained from the O_2 website, and plotted in figure 2.4 for the period 20th November 2001 to 7th March 2006. To complete the calculation, *unlevered* figures should be used, adjusting for the level of gearing over the period. We were not able to obtain detailed data on gearing, however. Hence, for the current exercise, we use raw share price data. The annualised percentage rate of change in these data is 29.17%; the annualised standard deviation is 43.51%.

2.41 In order to relate these figures to the drift and volatility of traffic volumes, we need to relate the dynamics of the MNO's value to the dynamics of traffic volumes. From Ito's lemma,

$$dV = \frac{\partial V}{\partial X} dX + \frac{1}{2} \frac{\partial^2 V}{\partial X^2} (dX)^2.$$

We are concerned only with the stochastic components of dV and dX, and so can ignore the second order term. That implies that

$$\mu = \Omega(X)\mu_V, \qquad \sigma = \Omega(X)\sigma_V$$

where

$$\Omega(X) \equiv \frac{V(X)}{X\frac{\partial V}{\partial X}}.$$

Our previous calculations have given us an expression for $V(X, K)$ (see equation (2.8)). Things are particularly simple when $X \geq K$; in this case, $\Omega(X) = 1/\beta$. This then means that

$$\frac{\mu}{\sigma} = \frac{\mu_V}{\sigma_V} = 0.6705. \tag{2.18}$$

2.42 This means that we cannot derive separate estimates of μ and σ: they are coupled by equation (2.18). If, however, we specify an estimate for μ, the annual percentage change in traffic volumes, then equation (2.18) supplies us with the corresponding estimate of σ, using share price data (which, by hypothesis, is forward-looking). We use the traffic forecasts in the latest LRIC model to estimate μ. The average annual percentage growth rate over the period 2005/06–2039/40 is 9.5%. Equation (2.18) then implies that the traffic volatility parameter is 14.1%.

2.43 Other parameters in table 2.1 are estimated in a rough-and-ready way. The (nominal, pre-tax) retail price p is set at 10ppm, based on figures from the 2003 Competition Commission report for total origination revenues and traffic minutes for the four operators. Marginal costs of origination and termination are set to zero (since they are likely to be small). The figure for f is taken from the 2003 Competition Commission report, and is intended to be an average for all fixed networks. The cost of capital ρ is taken as 11.3% in pre-tax, real terms. We use an inflator, γ, with a central value of 2.5%, so that the central nominal pre-tax cost of capital is 13.8%.

2.44 κ is a tricky parameter to estimate accurately. One method is to base the estimate directly on the LRIC model, using capital expenditure and the volume of forecast traffic for each year over the period 2006–2021. In order to match the outcome of real options model as closely as possible to current calculations, we choose instead to set κ so that the termination charge with deterministic traffic growth, given in equation (2.4), is roughly equal to the average recommended charge over the next review period, of about 6ppm. This then implies that $\kappa = 43.5$.

2.45 Finally, we allow operators' margin per unit of traffic to decline over time, at an annual rate equal to the deflator γ. A full treatment of a time-varying margin is very complicated;[5] We approximate this factor by adjusting downwards the annual percentage growth rate of traffic, so that the *effective* growth rate is $\mu - \gamma$.

2.46 With these parameter values, the termination charge (per unit of traffic) for the model without uncertainty is 6ppm (by design). Recall that the termination charge (per unit of traffic) for the model with uncertainty is

$$\frac{p - c^O - c^T + \left(\frac{1-a-b}{2}\right)(p - 3f - c^O) - (\hat{\beta} - 1)\rho\kappa}{a + b}.$$

At the central inflator figure of 2.5%, this gives a termination charge of 8.14ppm, some 36% higher than the corresponding charge under certainty. This higher value reflects directly the need to reward investing firms for the risk that irreversibility presents. The reward is not a 'monopoly rent', but a necessary increment to the standard LRIC to ensure efficient investment.

2.47 We have performed a sensitivity analysis for these calculations. The estimates are most sensitive to the parameters μ, σ and ρ (and particularly the gap $\rho - \mu$). This sensitivity is caused by the simplicity of the model, in which a high degree of linearity is assumed. For example, the profit function in equation (2.6) is a linear function of traffic, since we have not modelled demand in a detailed way. A consequence of this feature is that the termination charge under uncertainty depends linearly on the uncertainty factor $\hat{\beta}$. Since this factor is sensitive to parameter values (see figure 2.2, for example), so is the implied termination charge.

2.48 As a result, and given the approximate nature of the exercise, we want to emphasise the following main points (rather than a specific figure):

- Real options can be taken into account within the broad framework of the existing LRIC model.
- Real options can have a significant quantitative effect on the termination charge that should be applied.

[5] It turns the ordinary differential equation (2.7) in to a partial differential equation, in which the value function $V(\cdot)$ depends on both the state and time.

- At the central parameter values that we have used, real options increase the effective LRIC of termination of MNOs. The increase arises in order to yield adequate returns on investments that are irreversible and inherently risky, so that investment occurs efficiently.

2.4 Summary

2.49 Our primary aim in this chapter has been to indicate how the existing LRIC model can be adapted to take into account the real options that are inherent in investment decisions for mobile networks.

2.50 Our model works as follows:

2.50.1 The primary driver (i.e., the 'state' variable) is the volume of traffic.

2.50.2 The existing LRIC model treats traffic volumes as changing *deterministically*, and assumes that investment decisions follow traffic volumes. (Any lags that occur are due to the 'lumpiness' of network capacity.)

2.50.3 In contrast, we assume that traffic volumes vary *stochastically*: future changes in volumes are inherently uncertain. This uncertainty, combined with the irreversibility of investment, means that MNOs will not invest immediately when traffic volumes rise. Instead, they will wait until volumes are sufficiently high, relative to current capacity, before investing. This investment delay is entirely *efficient*.

2.50.4 We determine when and how much investment will occur.

2.50.5 Given the simple structure of our model, that allows us to compute an LRIC, and hence a regulated termination charge, per unit of capital. These are calculated to ensure that MNOs face efficient incentives towards investment.

2.50.6 Estimates of the model parameters indicate that allowing for real options raises the regulated termination charge by around 35%.

Chapter 3

The option for flexibility

3.1 In the previous chapter, we considered when and how MNOs will invest in network capacity when faced with irreversibility and uncertainty. Since there were no fixed costs to investment, we argued that investment, when it occurs, increases capacity up to current traffic volumes.

3.2 It is more realistic, however, to allow for fixed costs to investment. That is, when network capacity is expanded, there are costs that have to be borne by the MNO that are independent of the scale of investment. For example, when a new site is prepared, there are labour and other costs that have to be incurred regardless of the number of masts and the amount of capacity installed at the site.

3.3 An immediate implication of these fixed costs is that MNOs may, *efficiently*, over-invest in capacity. When traffic volumes rise to hit the investment boundary, the investment that is triggered brings capacity *above* the current volume of traffic. This is illustrated in figure 3.1. (The exact amount of investment can be determined using the approach described in the appendix.)

3.4 This behaviour can be viewed as a form of *flexibility*. Traffic volumes vary stochastically. It is not optimal or even possible, given irreversibility and uncertainty, simply to match capacity to traffic at each moment. If traffic volumes fall, then it is not possible to sell capacity and recover the cost of investment. If traffic volumes rise by a small amount, the fixed cost of investment (as well as real options considerations) mean that it is efficient to wait before investing in capacity. Once traffic volumes hit the investment boundary, then it is efficient to bring capacity *above* the current traffic volume, to economise on future fixed investment costs. The extra capacity gives the network the flexibility to take

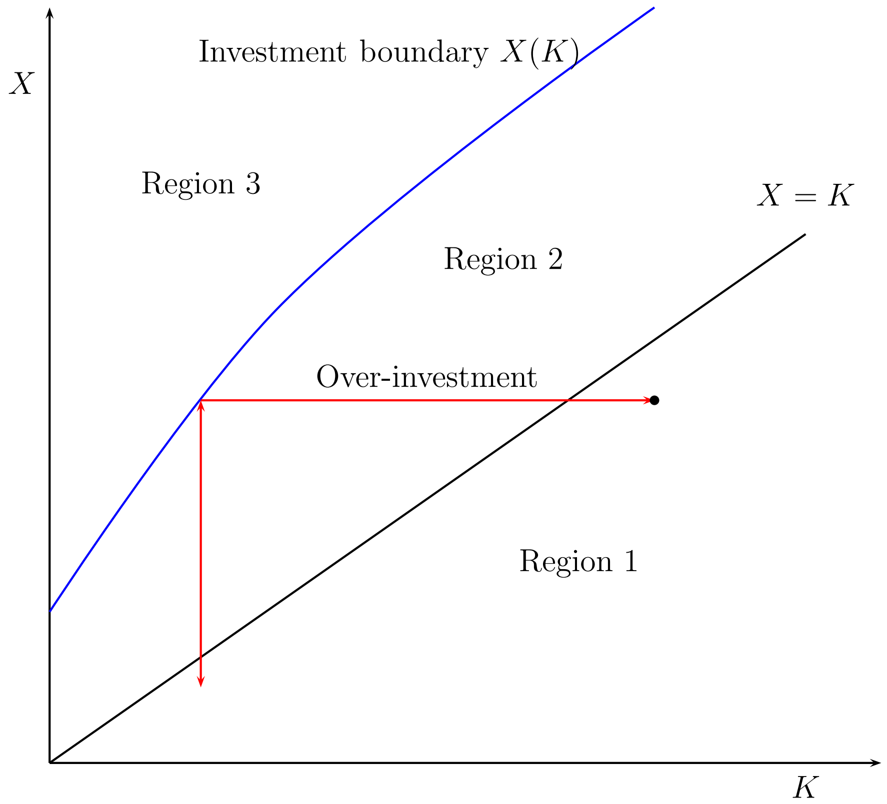

Figure 3.1: Over-investment for flexibility

on additional traffic without having to incur more fixed investment costs.

3.5 Equation (2.13) tells us that the investment boundary $X(K)$ increases in the size of the investment increment $K' - K$. This make sense: when optimal investment calls for a greater increment in capacity, then, with irreversibility and uncertainty, the traffic volume that triggers investment should be higher. Following this fact through the calculations in the previous chapter shows that, as a consequence, the regulated termination charge $\bar{t}$ should also rise. The logic is simple. Efficiency calls for MNOs to wait for higher traffic volumes before investing, and to make larger capacity increments when investing. This effectively exposes the MNOs to additional risk, which has to be reflected in the regulated charge. The point of raising the termination charge is not to be, in some sense, 'fair' to the MNOs. Rather, it is to ensure that the MNOs face efficient incentives towards investment.

Chapter 4

Conclusions

4.1 Our objective in this report has been to show how the current LRIC model for mobile termination can be adapted to incorporate the real options that arise when investment is irreversible and subject to uncertainty.

4.2 We have followed the current structure of the LRIC model by treating traffic volumes as the fundamental driver. We depart from the current LRIC model by assuming that traffic volumes vary stochastically. This, we have shown, has major consequences for investment behaviour, LRICs, and the level of the regulated termination charge.

4.3 We have considered two different types of options created when traffic volumes are stochastic. They relate to the option of when to invest; and the option of flexibility (or how much to invest).

4.4 In both cases, our objective has been to determine the form of *efficient investment*. We show that

4.4.1 MNOs will wait until traffic volumes are sufficiently high before investing.

4.4.2 When investment occurs, it can lead to over-capacity: the investment increment exceeds current traffic volumes.

Both types of behaviour are efficient. Moreover, the regulated termination charge should be set to ensure that MNOs behave in exactly this way.

4.5 We show in some detail how the regulated termination charge should be set to take into account the real option of when to invest. The termination charge must be increased from the level that ignores real options, in order to reflect the risk involved in irreversible, uncertain investments. A rough calibration of the model indicates that the increase in the termination charge is around 35%.

4.6 More important than this approximate quantitative effect is the framework that is developed to incorporate real options in regulatory decision for mobile networks.

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Chapter 5

Appendix: Optimal investment under uncertainty

5.1 Three conditions determine when and how much investment occurs:

$$V(X_t, K) = V(X_t, K') - \kappa(K' - K) - F; \tag{5.1}$$

$$\frac{\partial V(X_t, K')}{\partial K} = \kappa; \tag{5.2}$$

$$\frac{\partial V(X_t, K)}{\partial X} = \frac{\partial V(X_t, K')}{\partial X}. \tag{5.3}$$

These are, respectively, value matching (VM), optimal investment (OI), and smooth pasting (SP) conditions. $F \geq 0$ is the fixed cost of investment, which is unrelated to the scale of investment $K' - K$. The OI condition supposes that there is a non-zero but finite amount of investment. If

$$\frac{\partial V(X_t, K')}{\partial K} < \kappa$$

then no investment occurs (the marginal benefit from investment is outweighed by the marginal cost). If

$$\frac{\partial V(X_t, K')}{\partial K} > \kappa$$

then investment does not stop at K': capacity is increased to the point at which equality occurs between the marginal benefit from investment and the marginal cost.

5.2 We know that the value function $V(X_t, K)$ takes the form

$$V(X_t, K) = \begin{cases} A(K)X^{\beta} + \frac{\theta}{\rho-\mu}X_t & X_t < K, \\ A(K)X^{\beta} + \frac{\theta}{\rho}K & X_t \geq K. \end{cases} \tag{5.4}$$

5.3 Suppose first that $F = 0$ (there are no fixed costs of investment). Is it possible that $K' < X(K)$ i.e., that investment, when it occurs, leaves capacity below the current traffic volume? The VM, OI and SP conditions would then be

$$A(K)X(K)^{\beta} + \frac{\theta}{\rho}K = A(K')X(K)^{\beta} + \frac{\theta}{\rho}K' - \kappa(K' - K); \quad (5.5)$$

$$\frac{dA(K')}{dK}X(K)^{\beta} + \frac{\theta}{\rho} = \kappa; \quad (5.6)$$

$$\beta A(K)X(K)^{\beta-1} = \beta A(K')X(K)^{\beta-1}. \quad (5.7)$$

The third equation implies that $A(K) = A(K')$. The first equation then implies that $\theta/\rho = \kappa$ (which of course cannot hold in general). The second equation then gives the marginal return from investment as

$$\frac{dA(K')}{dK}X(K)^{\beta}.$$

This is zero (and hence investment non-zero but finite) if and only if $dA(K')/dK = 0$ for all K'.

5.4 In summary: optimal investment will be such that $K' < X(K)$ if and only if $\theta/\rho = \kappa$. Since θ, ρ and κ are parameters of the model, this is a very special case that will not arise in general. In fact, in the model calibrations in section 2.3, we find that $\theta/\rho = 78$ while $\kappa = 30$.

5.5 This leaves the other case, of $K' \geq X(K)$: investment occurs so that capacity at least matches the current volume of traffic. In the absence of fixed costs of investment ($F = 0$), it cannot be optimal for investment to bring capacity *above* the current volume of traffic: the additional capacity would earn no immediate return; and, in the event that the volume of traffic rises, the MNO can invest with constant returns to scale. Hence the only case to consider, when there are no fixed costs to investing, is $K' = X(K)$.

5.6 When there are fixed costs to investing, the three conditions are

$$A(K)X(K)^{\beta} + \frac{\theta}{\rho}K = A(K')X(K)^{\beta} + \frac{\theta}{\rho - \mu}X(K) - \kappa(K' - K) - F; \quad (5.8)$$

$$\frac{dA(K')}{dK}X(K)^{\beta} + \frac{\theta}{\rho} = \kappa; \quad (5.9)$$

$$\beta A(K)X(K)^{\beta-1} = \beta A(K')X(K)^{\beta-1} + \frac{\theta}{\rho - \mu}. \quad (5.10)$$

These can be solved to give the optimal K', which will be greater than $X(K)$ due to the fixed cost F.